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Polynomial Chaos Expansion in Bio-and Structural Mechanics

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Preface

This monograph is my doctoral thesis that was written within cotutelle programme in a partnership between Gdańsk University of Technology (Poland) and Institut National des Sciences Appliquées - Centre Val de Loire (France) and defended on October 12, 2018 [173]. Literature has been updated with a couple of recently published articles.

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Contents

Symbols

α_{aw}	orientation angle of the abdominal wall in lateral zone
α_{orient}	orientation angle of the surgical mesh
$\bar{\sigma}_{max}$	mean value of maximum principal stresses
$\bar{\sigma}_{min}$	mean value of minimum principal stresses
$\boldsymbol{\xi}$	standard input vector
δ_i	angular change in position of support
Δ_p	displacement of the cable edge
λ_i	eigenvalue of C
$\mathbf{A}^\top\mathbf{A}$, $\mathbf{A}^\top\mathbf{WA}$	information matrix
$\mathbf{C}$	correlation matrix
$\mathbf{X}$	random input vector
$\mathbf{Y}_{ex}$	vector of exact solutions
$\mathcal{A}$	truncation set
$\mathcal{H}_{\mathbf{X}}$	support of input vector $\mathbf{X}$
$\mathcal{H}_{X_k}$	support of random variable X_k
$\mathcal{LN}(\mu_{lni}, \sigma_{lni})$	lognormal distribution
$\mathcal{M}$	computational model
$\mathcal{M}^{PC}$	polynomial chaos metamodel
$\mathcal{M}_0$	mean
$\mathcal{M}_i(X_i)$, $\mathcal{M}_{i,j}(X_i, X_j)$, $\mathcal{M}_{1,2,\ldots,M}(\mathbf{X})$	terms of ANalysis of VAriance decomposition
$\mathcal{N}(\mu_i, \sigma_i)$	normal distribution
$\mathcal{T}$	isoprobabilistic transform
$\mathcal{U}([a, b])$	uniform distribution
$\max \sigma_{max}$	maximum value of maximum principal stresses
$\min \sigma_{min}$	minimum value of minimum principal stresses
μ_{cj}	coefficient of friction between wooden logs
$\Psi_{\boldsymbol{\alpha}}(\boldsymbol{\xi})$,$\Phi_{\boldsymbol{\alpha}}(\mathbf{X})$	multivariate polynomial basis
φ_i	eigenfunction of C

Ξ	design of experiment
A	cross sectional area of the cable
$a_{\boldsymbol{\alpha}}$	coefficients of the polynomial chaos expansion
C	covariance function
D	variance
$D_{i_1,\ldots,i_s}$	partial variance
E	elastic modulus of the cable
E_1, E_2	elastic moduli of surgical mesh in two directions
E_1^{aw}, E_2^{aw}	elastic moduli of abdominal wall in two directions
E_L	Young's modulus of wood in longitudinal direction
$Err_{\%}$	reference error
Err_s	error in calculating Sobol' indices
$Err_{PC\%}$	error of the PC metamodel
$F(\mathbf{Z}, \omega)$	random field
$f_{\mathbf{X}}$	probability density function of the input $\mathbf{X}$
f_{X_i}	marginal distribution of variable X_i
g	distributed load in the cable model
G_{12}^{aw}	shear modulus of abdominal wall
H	horizontal reaction in the cable model
H_0	initial force in the cable
H_n	Hermite polynomial
k_{aw}	stiffness of elastic foundation
k_f	stiffness of elastic springs
l	span of the surgical mesh/cable
L_0	initial length of the cable
L_n	Legendre polynomial
l_s	overlap of the surgical mesh onto the fascia
M	dimension of the input vector $\mathbf{X}$
N_{KL}	number of terms of Karhunen-Loéve expansion
N_{MC}	number of simulations in Monte Carlo method
$NRMSE$	normalized root mean square error
P	size of the polynomial chaos basis
p	polynomial order
p_{ia}	intraabdominal pressure
R_i	reaction force in i-th support

r_i	radial change in position of support
R_{max}	maximum reaction force
$RMSE$	root mean square error
S_i^{local}	local sensitivity index
$S_{i_1,\ldots,i_s}$	Sobol' sensitivity index
S_i^{Tot}	total Sobol' sensitivity index
t_i	forced displacement of i-th support
u_{aw}	displacement of point in healthy abdominal wall
u_i	displacement of the centre of the implant in the global model
u_{max}	maximum deflection of the implant
w	weight function
Y	model response, quantity of interest
DoE	abbreviation of Design of Experiment
FE	abbreviation of finite element
LHS	abbreviation of latin hypercube sampling
MC	abbreviation of Monte Carlo
PC	abbreviation of polynomial chaos
PDF	abbreviation of probability density function
QoI	abbreviation of quantity of interest
SA	abbreviation of sensitivity analysis
UQ	abbreviation of uncertainty quantification

Chapter 1

Introduction

The motivation of this study is the need for a mechanics-based approach to support the treatment of ventral hernia to help surgeons in solving the problem of hernia recurrences. Mathematical models are created to predict the mechanical behaviour of the implant-abdominal wall system and they can be used in the optimization of ventral hernia repair parameters. However, challenges such as the uncertainty related to natural variability of abdominal tissue mechanics and difficulties accurate measurement of material model parameters may occur in the modelling. Therefore, this study concerns an application of uncertainty quantification methods in the models of the implant-abdominal wall system.

1.1 Ventral Hernia

A ventral hernia is a bulge of tissues through a gap in the muscalo-fascial system. The hernia defect can be congenital, develop over time as a result of muscle weakness or be caused by trauma. Nowadays hernia commonly occurs at the place of an incision after other abdomen surgery (incisional hernia). In the study of Bensley et al. [10] hernia developed in 12% of patients after major abdominal surgery and in 3.3% after a laparoscopic operation. In France alone around 13 000 incisional hernia repairs are performed each year with an annual cost of around 84 million euros when estimated indirect cost related to sick leave etc. are included [57]. According to Skalski et al. [157], around 13 000 people are operated due to abdominal hernia every year in Poland.

According to Muysoms et al.[129], ventral hernias are classified based on the localisation, size and information on previous repairs. Primary hernia in terms of the location can be epigastric, umbilical, spigelian and lumbar. Incisional hernias, owing to their higher diversity, are divided into more subgroups: subxiphoidal, epigastric, umbilical, infraumbilical, suprapubic, subcostal, flank, iliac and lumbar. As shown in the previous work [178], the developed mathematical models can be applied to different locations of hernia.

The treatment of ventral hernia is usually carried out by surgical intervention. An implant in the form of a surgical mesh is connected by the surgeon to the abdominal wall to cover the defect. It can be performed by an open or laparoscopic operation. Laparoscopic ventral hernia repair (LVHR) is less invasive and is believed to be superior to open repair in terms of short-term results [143, 151]. Although a smaller number of postoperative complications were observed in patients treated by the laparoscopic method, the hernia recurrence rate is similar for both methods. Meshes for LVHR are typically made from polypropylene, polyester or expanded polytetrafluoroethylene [47]. It is desirable to reduce the number of hernia recurrences and pseudo-recurrences related to excessive bulging of the mesh. An increase of efficiency of hernia repair would have not only a clinical impact, but also a societal and economical one. It has been estimated that reduction of the recurrence rate only by 1% would save 32 million dollars just in the US [141]. Despite a number of studies, there is no consensus on the material and type of fixation which should be used in hernia repair [18].

Brown and Finch [20] wrote a medical review on surgical mesh choice which also described the history of surgical meshes as implants in hernia repair. The use of surgical meshes to reinforce the