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Fundamentals of classical and analytical mechanics

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CONTENTS

Preface

The primary intention of Author of the present book was to draw up a monographic picture of the contemporary attempts used to describe the Newtonian and Lagrangian Mechanics. But concurrently, the book can be recognized as a supplementary educational material, useful for the graduate courses in Mechanics taken by students majoring in Mechanical Engineering, Civil Engineering or Physical Science. The Faculty of Mechanical Engineering and Ship Technology at Gdansk University of Technology offers two intermediate-level undergraduate courses of mechanics, followed by an additional higher-level graduate course. And therefore, among the students, the book could be useful primarily for students of the higher-lever graduation, since it evolved from the Author's experiences and from his lecture notes used in the abovementioned more advanced one of the courses of mechanics. It has also some consequences in arrangement of structure of the present book. But we have to underline it, from the early beginning, the book was intended to express the subject wider and deeper that a textbook dedicated to a course. It is strongly recommended to the readers and the students, who intend to read this book, to take a contact with some elements of theory of Mechanics and of mathematics, previously. With a limited amount of exceptions, the students who took the undergraduate courses had obtained the experience necessary to read this book. The experience is not a homogenous one, but vital, since they had an occasion to follow the introductory courses in Mechanics at the undergraduate level. Moreover, many of them had also followed advanced courses of physics, or at least, they had followed general calculus-based courses of physics at the university. Thus, we may assume that a bit of a solid background knowledge in Mechanics already exists in their cases. In addition, certain familiarity with basic concepts of mathematics is also vital for reading this book. We should recommend that courses in differential/integral calculus, differential equations and matrix algebra should precede the reading of this book and eventual courses based on the contents of the book. The readers should understand such concepts as Taylor expansions in polynomial series for multivariate functions; partial derivatives; the chain rule of differentiation. Some elementary knowledge about vector and matrix calculus is also worth for recommendation.

Following the idea of monographic presentation, a certain conclusion should be drawn, and we fully agree with that opinion, we have included a bit too much of theoretical information in this book in compare to the ones that can be rationally presented in a semester of a graduate course. But it is an intentional decision, since the intention was to write a standalone monographic book. We hope, the book can be used as the supplementary material. It should help to construct and to supervise the course, but also it should help to extend the fundamental knowledge obtained during the course with some additional aspects. It is the task of the teachers and of their experience to select these parts and

elements of the book which they feel as fundamental or constitutive for the presented by them course in mechanics.

If selected as an element of a course, the book may not be considered as a standalone element used in the learning process. The process will be much more effective, when the book will be used as a part of synchronised actions, which all involves the students in their active learning process. Standard lectures with some active visual presentations and derivations of formulas, dialogs within the groups of students, seminars and exercises are essential for the educational success. The pressure to present as much as possible of the theoretical information in the lecture can be ineffective, since it tends to reduce all the potential introspective discussions to a minimum. Reading the book alone will not fill this gap, also. Students tend to read texts rather superficially, "from the outside" or "on the surface", only. They tend to accept the facts, but without any deeper reflection about the conditions and/or applications. And therefore, the afore-mentioned more sophisticated process, the one obtainable from merging of lectures, exercises and seminars, has its fundamental meaning in the educational process resulting of the course associated with in the book.

A short synopsis of the subsequent chapters follows:

- Chapter 1. Few words of introduction focusing on the domain, branches and the primary assumptions of mechanics;
- Chapter 2. A brief introduction to the fundamental concepts and principles of algebra of vectors;
- Chapter 3. Kinematics of particles, mainly focused on non-Cartesian coordinates used in mathematical descriptions of kinematics;
- Chapter 4. Dynamics of particles; focused on Newton's laws of motion; motion of particles in one and in three dimensions; dynamics expressed in inertial and non-inertial frames of reference; the principles of linear momentum and of angular momentum of particle; work and energy principles;
- Chapter 5. Kinematics and dynamics of spherical motion of rigid bodies; Euler's angles; angular velocities and angular accelerations; conical surfaces of angular velocity vectors, steady precession, angular momentum of bodies in their spherical motion, Euler's equations of dynamics for bodies in their spherical motion; kinetic energy formulae;
- Chapter 6. Kinematics and dynamics of general motion of rigid bodies; linear velocity and linear acceleration of points that belong to the same body; linear and angular momentum of rigid body; Newton-Euler's dynamics equations; kinetic energy;
- Chapter 7. Simplified theory of gyroscope;
- Chapter 8. Mechanics expressed with use of Lagrange's formalism; constraints and the constraint equations; main principles used in classifications of the constraints; generalized coordinates; virtual displacements and consequences of the virtual work principle; virtual velocities and the virtual

power; generalized forces and the generalized equation of dynamics (d'Alembert's principle); two types of Lagrange's equations; selected alternative principles of equilibrium.

Few advices to students who intends to read the book

Dear Readers, learning and understanding Mechanics is an active and continuous process. It requires in a drop of experiences and repetitions. When you were learning to swim, ride a bicycle or to play the guitar, You have not expected that the results would come over a day. To learn this, you had practiced the related technique until it became a kind of your second nature. Falling off the bicycle was not a tragedy. A false note of the guitar wasn't it, also. Truly speaking, it was a kind of inspiration to practice more. Learning Mechanics is exactly the same kind of activity. To learn it, the knowledge of elements of Mechanics must become a part of your intuitive nature. Not all the aspects become evident after a single reading. But it is the same, like with the fall off the bicycle. It should be an inspiration to practice more. Anyone, even me, had to practice hard in order to understand mechanics. And furthermore, more you know - more of the aspects arise again and you would like to reconsider them one more time. You should understand and accept it, the learning is an active process. One thing is to watch your lecturer solving a problem or developing a formula, when all the steps seem to be logical ones. But, it is a quite another thing and difficulty to try to solve the problem on your own. Learning is not an "art of appreciation". It is a "made it yourself" type of activity. The key of the success is not in the classroom. It is outside of the classroom. It is the way you study alone. The last minute, or the "all night", studies are the straight ways to a self-destruction and to defeats during exams. Thus, you are strongly recommended against to try to learn this way. You should reserve a regular time for studying the material. You should avoid any unnecessary distractions when reading. You should think about theories or problem earlier that a day before your knowledge is evaluated. Try to mark all the aspects and the theories you do not understand, and try to read again the material few days after. Discuss the problems with the students and with the lecturers. Do not hesitate to ask the others. Read other books. Try to find some alternative descriptions of the problems which you have not understood. Try to memorize what notations are used and what these notations mean. We have observed that lack of the familiarizations with the used symbols is one of the biggest problems during the discussions. To memorize them, it requires in time and in practice. You should repeat their meaning, until their interpretations will not become automatic. And when you do not understand the aspects which you are reading, try to construct your own simple and elemental examples. Never let the lack of the understanding remains – ask someone. When you discover a set of problems which you cannot understand, do not assume they are clear to everyone, except you. Ask the other students, it can help you to see that some of them are also not able to understand it, and that the others have spent a lot of time to understand.

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First and foremost, I would like to address may gratitude to all the students who took my course over last years, all these years when the form and the contents of the book

were developed. They gave me occasion to get the experience and they were the biggest motivation to write the book, also.

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