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**Nonstandard Equivalence Scales  
and their Applications  
for European Union Countries**

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*To my grandchildren*

*Emma, Łucja, Kosma and Ignacy*



*Make things as simple as possible, but not simpler*

*Albert Einstein*



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# Introduction

The strand of theoretical research called *the distributional analysis* assumes a well-defined distribution of incomes or expenditures ((hereafter treated as exchangeable) in a homogeneous population of comparable units.<sup>1</sup> Many theoretical models of income distributions have been developed (see, e.g. Kleibler and Kotz, 2003).

However, a confrontation of these theoretical models with actual income data faces severe difficulties. Available statistical data come from household budget surveys where statistical units are households, not individuals. Households differ in many nonmonetary characteristics, e.g., size, demographic composition, the presence of disabled household members, and the like. We shall refer to these characteristics as *household needs*, for short.

Standard practice is to divide household incomes by an equivalence scale, a factor depending on the needs, to obtain *the equivalent incomes* which, by construction, are comparable across income units. Equivalence scales are the tools for converting a heterogeneous population of households into a homogeneous population of welfare-equivalent units.

The traditional analysis of income distribution runs in two steps (Ebert, Moyes, 2003). In the first step, a reference household group and an equivalence scale are chosen. Usually, the reference group comprises households of single childless persons, although other specifications are possible. Then, the equivalence scale adjusts the actual incomes for individual groups of households. Standard inequality, welfare, and poverty measures are calculated for the adjusted income distribution in the second step. These stages are treated separately.

However, there are two serious reasons why the two-stage procedure is unsatisfactory. First, the homogenising stage does not provide precise results. Equivalence scales are part of a social valuation process. There is no single ‘correct’ equivalence scale for adjusting incomes (Coulter et al., 1992a). The second objection concerns the interdependence of the two stages. There is evidence that the results of distributional comparisons are sensitive to the choice of equivalence scale (Coulter et al., 1992a,b).

The aim of this study is twofold. First, we propose new methods of estimating equivalence scales. We call these methods *nonstandard* to distinguish them from the standard procedures based on the microeconomic theory of demand systems. Searching for the ‘as simple as possible’ method has been a leitmotiv of this study. Second, we estimate various forms of equivalence scales for 28 European Union (EU) member countries and four non-member European countries from 2004 to 2018.

This study represents the past several years of the author’s research on income distribution and draws heavily upon several of his published and forthcoming articles. The monograph format of this publication enables the presentation of many additional results, which usually cannot be placed in a typical paper due to limited space. It is noteworthy that material from these papers does not appear in precisely the same format as initially published; they have been edited to form a coherent whole.

This monograph consists of three parts. Part I, entitled “Microeconomic equivalence scales and data”, consists of two chapters. Chapter 1 offers a concise review of the standard microeconomic equivalence scales. Several shortfalls offset various indisputable advantages of the equivalence scales. This chapter also presents the most promising current theoretical alternatives to the standard paradigm of equivalence scales. The chapter entitled “Statistical data” describes the variables from the European Union Statistics on Income and Living Conditions (EU-SILC) database used in this study.

Part II, entitled “The estimation of the random equivalence scales”, comprises three chapters. Chapter 3, entitled “The random equivalence scales: a theoretical background”, offers a stochastic perspective on the deterministic

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<sup>1</sup> Throughout this monograph, we use the term ‘income’ rather than the ‘expenditure’ or ‘consumption.’ In a one period model, all these are the same, because there is no savings to put a wedge between income and spending (Pendakur, 2018).

concept of equivalence scales developed within the microeconomic theory. In the general populations of households, the standard equivalence scales and underlying utility functions become continuous random variables according to the limit theorems. The concept of random equivalence scales (RES, for short), defined within a stochastic framework, corresponds to real circumstances more adequate than the former equivalence scales.

Chapter 4 presents the estimates of RES for 32 European countries from 2004 to 2018. Here, household size, i.e. the number of household members, expresses household needs. We distinguish the welfare-dependent and welfare-independent versions of the RES using subjective welfare assessments made by households. Examining estimates of RES shows that an analyst may use welfare-independent instead of welfare-dependent RES at the cost of a paradox: the greater inequality aversion, the more significant concern for affluent people. It contradicts applied welfare economics' claim that the greater inequality aversion, the greater concern for poor people. We call this contradiction 'the Rawlsian paradox'. This chapter also analyses trends of the mean and inequality in the distributions of equivalent incomes adjusted by the RES.

Chapter 5 offers estimates of the RES when the number of adults and the number of children express household needs. The results also reveal the Rawlsian paradox. In other words, an analyst may substitute the welfare-dependent RES with welfare-independent RES, but he/she should be aware that the greater inequality aversion accompanies the greater concern for rich people.

Part III, entitled "The estimation of parametric equivalence scales", comprises three chapters. Chapter 6 offers "the minimum Gini method" of estimating the one-parameter Buchmann et al. (1988) equivalence scale. The method is based on the U-shaped relationship  $(\theta, G)$  between the parameter  $\theta$  of the equivalence scale and the Gini index. We apply the Lind and Mehlum (2010) test to check whether this relationship is U-shaped. This chapter also contains the estimates of  $\theta$  for European countries. We assume that the household size measures household needs.

Chapter 7 presents the estimates of two-parameter equivalence scales, namely the OECD scale (2013) and Cutler and Katz's (1992) scale for European countries. We apply two versions of the minimum Gini method for estimating the parameters of these scales: numerical and analytical.

Chapter 8 aims to establish empirical similarities and dissimilarities between equivalence scales presented in this study. For this purpose, we calculate fifteen versions of the European income distribution (EUID). Each version consists of the disposable incomes of all European households in a given year adjusted by a particular equivalence scale.

The last chapter offers a final summary.

All time series in this monograph are smoothed with the distance-weighted least squares (Netter et al., 1996). Appendices following the main text have numbers corresponding to the number of chapters to whom they belong.

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